

Math 30-1 - Permutations and Combinations Assignment

Name Solutions

1. A vehicle license plate consists of 3 letters followed by 4 digits. How many different license plates are possible if letters cannot be repeated, but digits can?

$$\begin{array}{ccc} \underline{26} & \underline{25} & \underline{24} \\ \text{letters} & & \end{array} \quad \begin{array}{cccc} \underline{10} & \underline{10} & \underline{10} & \underline{10} \\ \text{digits} & & & \end{array} \quad 156,000,000 \\ \text{license plates}$$

2. How many odd, four digit numbers can be formed from 2, 3, 4, 5, 6, 7 if digits can be repeated?

$$\underline{6} \quad \underline{6} \quad \underline{6} \quad \underline{3} \quad 6^3 \times 3 = \underline{\underline{648}}$$

3. How many arrangements can be made of the letters of SWEET if
 A. There are no restrictions
 B. The consonants are not all together

$$\frac{5!}{2!} = 60$$

$$\boxed{SWT} | EE$$

$$60 - \frac{3!3!}{2!} = \underline{\underline{42}}$$

- C. They start with exactly one E
 D. The first letter is a vowel and the last a consonant.

$$\underline{2} \quad \underline{3} \quad \underline{3} \quad \underline{2} \quad \underline{1} \\ \frac{2 \times 3 \times 3!}{2!} = \underline{\underline{18}}$$

$$\underline{2} \quad \underline{3} \quad \underline{2} \quad \underline{1} \quad \underline{3} \\ \frac{2 \times 3! \times 3}{2!} = \underline{\underline{18}}$$

4. How many arrangements can be made of the letters of the word PIANO if
 A. The vowels are together

$$\boxed{IAO} | PN$$

$$3!3! = \underline{\underline{36}}$$

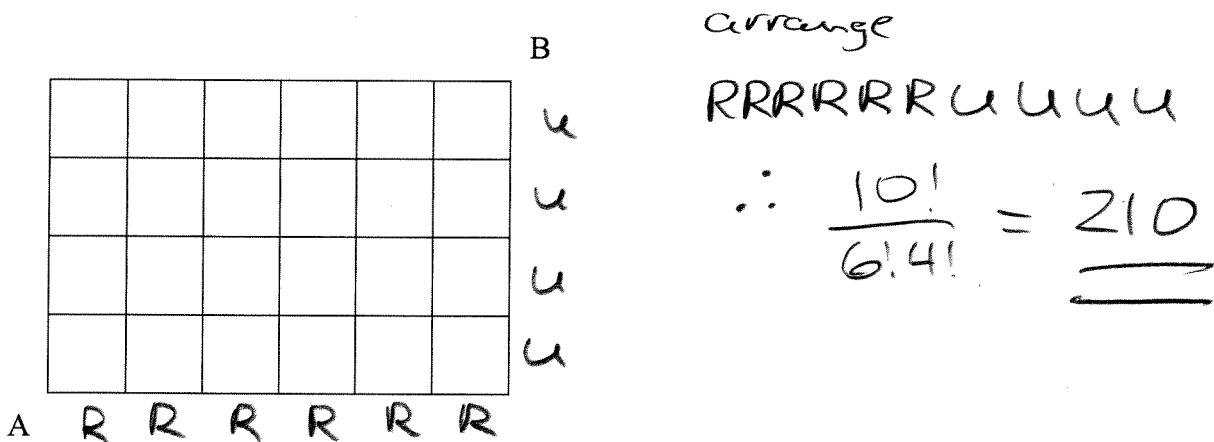
- B. The arrangements begin and end with a consonant

$$\underline{2} \quad \underline{3} \quad \underline{2} \quad \underline{1} \quad \underline{1} \\ 2 \times 3! \times 1 = \underline{\underline{12}}$$

8. How many 8's are there in the digits of the coefficient of the term containing x^6 in the expansion of $(3-2x^2)^9$.

$$\begin{aligned}
 T_{k+1} &= \binom{9}{k} (3)^{9-k} (-2x^2)^k \rightarrow x^6 \text{ so } k=3 \\
 &= \binom{9}{3} (3)^6 (-2x^2)^3 \\
 &= 84 \cdot 729 \cdot -8x^6 \\
 &= -489888 x^6 \quad \therefore \underline{\underline{4-8's}}
 \end{aligned}$$

9. How many pathways are there from A to B in the diagram if you must always move up and to the right?



10. In a jewelry store, there are various cabinets with different jewelry. In one cabinet, there are 10 different men's watches. In another cabinet, there are 8 different women's watches. Five watches are picked. In how many ways can these five watches be arranged if two of them come from the men's cabinet and three of them come from the women's cabinet.

$$\underbrace{10C_2 \times 8C_3}_{\substack{\# \text{ of ways} \\ \text{to get the} \\ \text{watches}}} \times \underbrace{5!}_{\substack{\text{arrange} \\ \text{them}}} = \underline{\underline{302400}}$$

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17